Quantum Field Theory

Exercises in preparation for the exam 3

Exercise 1: two real scalar fields

We will continue the exercise 1 of Homework 2. Consider a theory with two real scalar fields φ_1 and φ_2 and the following Lagrangian :

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi_1\partial^\mu\varphi_1 + \frac{1}{2}\partial_\mu\varphi_2\partial^\mu\varphi_2 + g\partial_\mu\varphi_1\partial^\mu\varphi_2 - \frac{m^2}{2}(\varphi_1^2+\varphi_2^2) - \frac{\lambda}{4!}(\varphi_1^2+\varphi_2^2)^2$$

a) For what values of g this represents a well defined QFT ? Find the masses of the physical particles. **This point you already computed in Exercise 1 of Homework 2, it is here for completeness.**
b) Compute the cross section for the scattering of two heavier particles into two lighter particles.

Exercise 2: scalar-fermions Lagrangian and large N

Consider N Dirac fields $\{\psi_a\}$, $a = 1, \ldots N$ and a scalar field ϕ.

- Assuming both ϕ and ψ have canonical kinetic terms, write the most general Lorentz invariant Lagrangian involving terms with canonical dimensions $d \leq 4$ which is symmetric under $U(N)$ rotations of the Dirac fields:

$$U_{ab}\psi_b(x), \qquad U_{ab} \in U(N).$$

  Is the most general Lagrangian also parity invariant?
  Hint: prove that mass terms of the kind $\bar{\psi}\gamma_5\psi$ can always be removed via a chiral rotation $\psi \to e^{i\alpha\gamma_5}\psi$.

- Supposing that ϕ is heavy enough and that the ψ_a have canonical Dirac mass term, compute the decay rate Γ of ϕ at tree level. What happens (and why) in the $N \to \infty$ limit?

Exercise 3: modified $O(2)$ model

Consider the following model of 2 scalar fields and a Dirac fermion:

$$\mathcal{L} = \frac{1}{2}\sum_{i=1}^{2}(\partial_\mu\phi_i)^2 - \frac{m^2}{2}\sum_{i=1}^{2}\phi_i^2 - \frac{\lambda}{4}\left(\sum_{i=1}^{2}\phi_i^2\right)^2 + \bar{\psi}(i\partial\!\!\!/)\psi - g\bar{\psi}(\phi_1 + i\gamma_5\phi_2)\psi.$$

- Find the global symmetries of the system.
  Hint: it might help to decompose explicitly the Dirac field in terms of Weyl spinors.

- Draw all Feynman diagrams contributing to the process $\psi\psi \to \psi\psi$ and write the matrix element.

- Compute the differential unpolarized cross section for the process $\psi\psi \to \psi\psi$ in the case $m = 0$.